[8] Y. S. Lee, "Mode compensation applied to parallel-coupled microstrip directional filter design," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-22, pp. 66–69, Jan. 1974.
[9] L. I. Maissel and R. Glang, *Handbook of Thin Film Technology.* New York: McGraw-Hill, 1975, ch. 16.
[10] S. Albin, "Electrical properties of Bi_2O_3," Ph.D. dissertation, University of Poona, Poona, India, 1975, ch. 4.
[11] S. Albin and K. Sathianandan, "Space charge limited current in Bi_2O_3 films," *Thin Solid Films*, vol. 24, pp. 340–341, Dec. 1974.
[12] H. A. Wheeler, "Transmission-line properties of parallel strips separated by a dielectric sheet," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 172–185, Mar. 1965.
[13] T. G. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.* (*1968 Symposium Issue*), vol. MTT-16, pp. 1021–1027, Dec. 1968.
[14] K. M. Johnson, "*X*-band integrated circuit mixer with reactively terminated image," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Microwave Integrated Circuits*), vol. MTT-16, pp. 388–397, July 1968.
[15] L. Holland, *Vacuum Deposition of Thin Films.* London, England: Chapman & Hall, 1956, p. 453.
[16] A. Farrar and A. T. Adams, "Multilayer microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-22, pp. 889–891, Oct. 1974.
[17] G. D. Vendelin, "Limitations on stripline *Q*," *Microwave J.*, pp. 63–69, May 1970.

Shuttle Pulse Measurement of Mode Coupling and Conversion

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Abstract—**The measurement of weak coupling between two guided modes, for which the usual TE_{10} setups cannot be directly used, is dealt with. This problem mainly arises in coupling between different waveguides and in overmoded waveguides. In both cases a representation by means of two coupled lines is chosen, described by a directional coupler scattering matrix.**

In addition to the well-known resonant Klinger method, a new one, in the time domain, is suggested. This new method makes use of a shuttle pulse test set and starts from the observation of the envelope of an output signal, which under suitable conditions shows zeros related to the coupling to be measured.

Experimental results, with frequencies up to 90 GHz and couplings as low as −40 dB, confirm the accuracy and the sensitivity of the new method.

I. Introduction

This short paper deals with the measurement of weak coupling between two guided modes, in reciprocal lossy junctions for which the classical insertion-loss method cannot be easily applied. This very often occurs in directional couplers between different waveguides and in overmoded circular waveguides. Assuming for the latter case that the coupling between wanted and spurious modes, due to unavoidable imperfections or to the design itself (in tapers, elbows, mode launchers, etc.), can be separately taken into account for each spurious mode, and neglecting backward waves, the junction can be represented by a four-port scattering matrix as

$$S = \begin{vmatrix} 0 & S' \\ \tilde{S}' & 0 \end{vmatrix}. \qquad (1)$$

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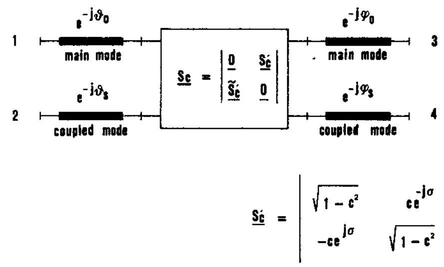

Fig. 1. Equivalent circuit of a coupled-mode junction.

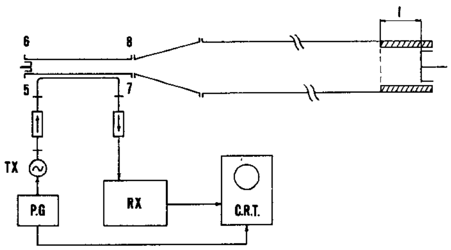

Fig. 2. Shuttle pulse setup used for conversion measurement.

Moreover, assuming a lossless coupling, S' becomes

$$S' = \begin{vmatrix} \sqrt{1-c^2}\, e^{-j(\theta_0+\phi_0)} & c e^{-j(\theta_0+\phi_s+\sigma)} \\ -\, c e^{-j(\theta_s+\phi_0-\sigma)} & \sqrt{1-c^2}\, e^{-j(\theta_s+\phi_s)} \end{vmatrix} \qquad (2)$$

where c and σ are the modulus and argument of the coupling, and θ, ϕ are complex parameters related to the main or wanted mode (subscript 0) and to the coupled or spurious one (subscript s). Under the preceding assumptions the junction can be represented as in Fig. 1.

When ports 2, 3, and 4 are short circuited, the reflection coefficient at port 1 becomes

$$\rho_1 = e^{-j2(\theta_0+\phi_0)} \frac{e^{-j2(\theta_s+\phi_s)} - T_2}{1 - T_1 e^{-j2(\theta_s+\phi_s)}} \qquad (3)$$

where

$$T_1 = 1 - c^2 + c^2 e^{-j2(\Delta\phi-\sigma)} \qquad (4)$$

$$T_2 = 1 - c^2 + c^2 e^{j2(\Delta\phi-\sigma)} \qquad (5)$$

and

$$\Delta\phi = \phi_0 - \phi_s. \qquad (6)$$

The evaluation of c can be performed by means of resonant methods [1]–[5] starting from measurements of ρ_1 [see (3)]. In some cases, as at millimeter wavelengths, a time-domain method, making use of a shuttle pulse test set, may be more convenient.

II. The Shuttle Pulse Method for Coupling Measurement

Let us consider the bilinear transformation of ρ_1 accomplished by means of the directional coupler of the shuttle pulse test set shown in Fig. 2. The transmission coefficient G, between the ports 5 and 7, can be expressed as

$$G = H - \frac{K}{1 + \rho_1 e^{-j2\psi}} \qquad (7)$$

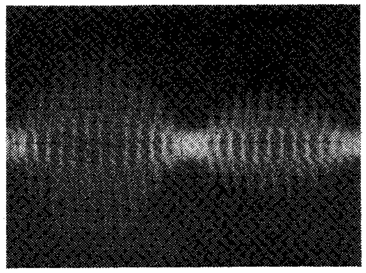

Fig. 3. Output pulses on the CRT.

where H, K depend on the directional coupler only and $-e^{-j2\psi}$ is the reflection coefficient looking into port 8. By using (3) and setting

$$\Delta\theta = \theta_0 + \psi - \theta_s \tag{8}$$

$$M = e^{j(\Delta\theta + \Delta\phi)} \tag{9}$$

$$X = \frac{1}{2}\left(T_1 M + \frac{T_2}{M}\right) \tag{10}$$

(7) becomes

$$G = H - K\frac{1 - T_1 M z^{-1}}{1 - 2Xz^{-1} + z^{-2}} \tag{11}$$

where

$$z = M^{-1} e^{j2(\theta_0 + \phi_0 + \psi)}. \tag{12}$$

For each input pulse, of limited bandwidth B centered on the measuring frequency f_0, a series of n pulses can be observed on the CRT of the shuttle pulse test set; the time interval between adjacent pulses is equal to $2\tau_0$, τ_0 being the overall group delay for the main mode.

If in the bandwidth B the coupling is constant and the $\Delta\phi$ and $\Delta\theta$ variations with frequency are negligible with respect to those of ϕ and θ, (12) may be written as

$$z \simeq e^{q} e^{j2\tau_0(\omega - \omega_0)} \tag{13}$$

where

$$q = |\theta_0 + \phi_0 + \psi + \theta_s + \phi_s|_{\omega_0}$$

and all the quantities but z in (11) may be thought frequency independent. Then, replacing in (13) $\omega - \omega_0$ by w in the limited bandwidth $W = 2\pi B$, (11) becomes the transmission coefficient G_w of the equivalent low-pass channel. Thus the amplitude of the nth output pulse is proportional to the modulus of the nth sample g_n of the complex envelope which corresponds to the inverse transform of G_w: thus

$$g_n = e^{-qn}[H\delta(n) - KU_n(X) - T_1 M U_{n-1}(X)] \tag{14}$$

where δ and U are the Dirac and second kind Chebyshev functions, respectively. Under the same hypotheses, (14) can be used up to a number of pulses

$$n \ll \frac{1}{2B(\tau_0 - \tau_s)} \tag{15}$$

τ_s being the overall group delay for the coupled mode.

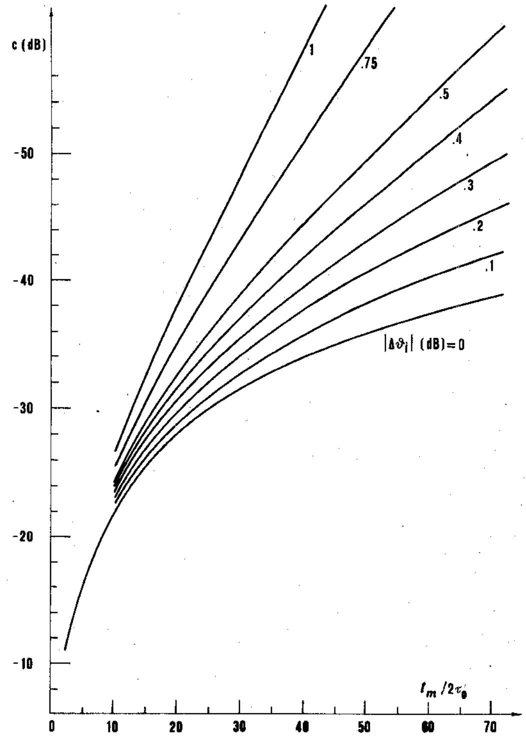

Fig. 4. Coupling versus time for the first zero of the output envelope, with $|\Delta\theta_i|$ as parameter.

Let us consider the envelope $g(t)$ of the samples g_n, obtained from (14) by setting $n = t/2\tau_0$. It becomes zero if, and only if

$$\arg(T_1) = -\arg(M) \tag{16}$$

and

$$\arg(T_1) = -\arg(T_2). \tag{17}$$

The first value t_0 that makes $g(t) = 0$ for $t > 0$ is given by

$$t_0 = \frac{2\tau_0}{\cos^{-1} X}\tan^{-1}\left(\frac{\sqrt{1 - X^2}}{|T_1 M - X|}\right) \tag{18}$$

assuming $\cos^{-1}$ and $\tan^{-1}$ between 0 and $\pi/2$. The other values are

$$t_i = t_0 + i\frac{2\pi\tau_0}{\cos^{-1} X} \quad (i = 1,2,3,\cdots). \tag{19}$$

Through the observation of the zeros of the envelope, very easy to accomplish by counting the output pulses on the CRT display shown in Fig. 3, it is then possible to evaluate the coupling c with a good accuracy, as will be shown in the next section.

III. The Shuttle Pulse Coupling Measurement

Equation (16) always can be satisfied by properly positioning the sliding short circuit in arm 6 which affects ψ [see (8), (9)] directly. The other equation (17) can be satisfied by positioning the short circuits in arms 3 and 4 of the junction in such a way to obtain $\Delta\phi_r - \sigma = k\pi/2$ making T_1 and T_2 both real [see (4), (5)]; furthermore, it should be noticed that $T_1 \simeq T_2$ in the case of negligible $\Delta\phi_i$, i.e., for slightly different losses in arms 3 and 4, regardless of $\Delta\phi_r$. In this latter case we get

$$X = |T_1| \cosh \Delta\theta_i \tag{20}$$

$$T_1 M - X = |T_1| \sinh \Delta\theta_i \tag{21}$$

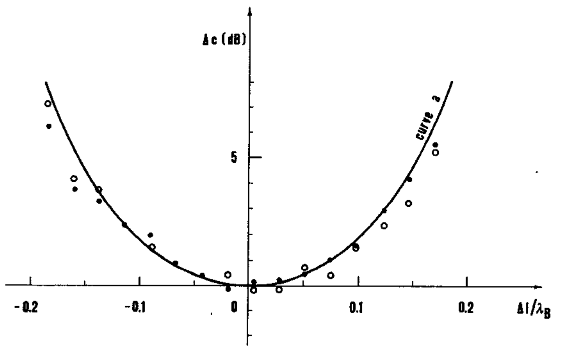

Fig. 5. Difference Δc between the theoretical value (-20 dB) and the measured one for the TE_{01}–TE_{02} conversion due to a diameter change; curve a shows the systematic error due to incorrect position (Δl) of the short circuit relative to the beat wavelength (λ_B); dots and circles refer to two series of measurements.

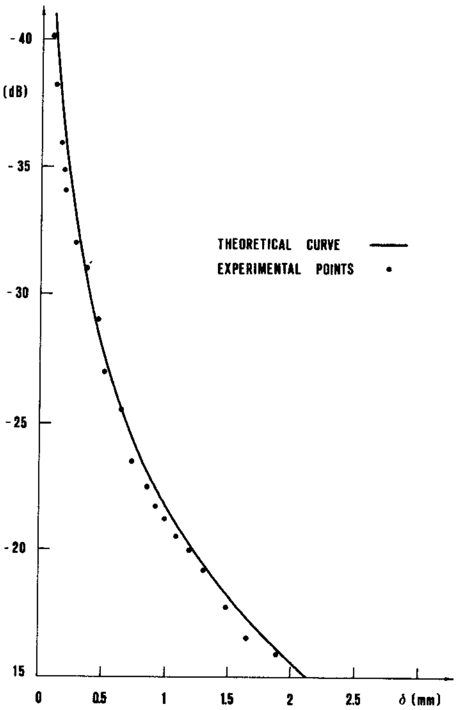

Fig. 6. TE_{01}–TE_{12} conversion versus an offset δ.

where $|T_1|$ is

$$|T_1| = \sqrt{1 - 4c^2(1 - c^2)\sin^2(\Delta\phi_r - \sigma)}. \tag{22}$$

Varying the relative positions of the short circuits in arms 3 and 4, varies $\Delta\phi_r$ between $\sigma + k\pi$, for which the coupling does not affect the envelope at all, and $\sigma + (2k + 1)\pi/2$, for which the coupling has the maximum effect, i.e., $|T_1| = 1 - 2c^2$ and t_0 reaches its minimum value t_m. This latter condition is best to perform coupling measurements. For a quick evaluation, Fig. 4 shows the coupling versus $t_m/2\tau_0$ obtained using (18)–(22) for typical $|\Delta\theta_i|$ values. It is worthwhile to mention that there is no difficulty in finding $|\Delta\theta_i|$ by the conventional use of the shuttle pulse test set. Moreover, if the two modes in arms 3 and 4 are propagating in the same waveguide, the condition $t_0 = t_m$ is obtained by any displacement of the common short circuit equal to a multiple of half a beat wavelength. With this technique the coupling mode can easily be identified.

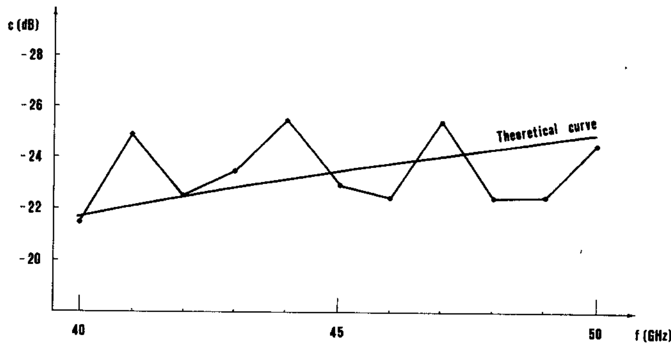

Fig. 7. TE_{01}–TE_{02} conversion versus frequency for a mirror corner.

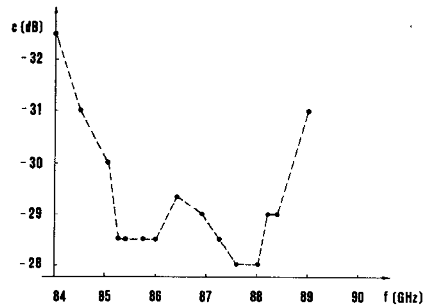

Fig. 8. TE_{01}–TE_{02} conversion versus frequency for a taper.

The sensitivity is limited by (15) or by the losses. As far as the method is concerned, the casual error increases as $|\Delta\theta_i|$ increases (see Fig. 4); systematic errors arise from $\Delta\phi_i \neq 0$ and from uncorrected positions of the short circuits: the former can be minimized by setting the short circuits in arms 3 and 4 of the junction as near as possible to the coupling region, while the latter is not critical [see (22)].

The accuracy of the measurement was tested with the setup shown in Fig. 2. The coupling between TE_{01} and TE_{02} modes in a ID 51-mm circular waveguide was intentionally introduced by means of a diameter change, with a theoretical coupling of -20 dB. In Fig. 5, the differences between theoretical and measured values are plotted for two series of measurements at 50 GHz, obtained by displacing the common short circuit in arms 3 and 4. The results confirm the above mentioned low sensitivity with respect to the systematic error (curve a), and show very little casual errors. The measured $|\Delta\theta_i|$ was 0.10 dB.

The sensitivity of the setup was tested by measuring the TE_{01}–TE_{12} conversion due to a variable offset δ. The results, plotted in Fig. 6 ($|\Delta\theta_i| = 0.12$ dB, $f_0 = 50$ GHz), show a sensitivity better than -40 dB, comparable with those recently obtained [3], [4].

In Figs. 7 and 8 measured values of the TE_{01}–TE_{02} conversion due to a mirror corner (in the 40–55-GHz band) and due to a taper (in the 84–90-GHz band) are presented.

References

[1] Y. Klinger, "Measurement of spurious modes in over-moded waveguides," *Proc. Inst. Elec. Eng.*, vol. 106B, Suppl. 13, pp. 89–93, 1959.
[2] F. Valdoni, "Misure a risonanza dell'accoppiamento tra modi diversi," *N.R.N.*, vol. 22, pp. 440–449, July–Aug. 1973.
[3] S. Shindo, "Tapered waveguide higher mode measurement by a resonant method," *E.C.L. Rev.*, vol. 22, pp. 139–144, Jan.–Feb. 1974.
[4] B. S. Seip and L. W. Hinderks, "A computerized Klinger cavity mode conversion test set," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 873–878, Oct. 1974.
[5] V. Rizzoli, "Resonant measurement of even and odd mode propagation constants in coupled microstrips," presented at the IEEE Microwave Theory Tech. Int. Microwave Symp., Palo Alto, CA, May 1975.